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Review

Some open problems in granular matter mechanics

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Abstract

Granular matter is a large assemblage of solid particles, which is fundamentally different from any other type of matters, such as solid and liquid. Most models presented for granular matter are phenomenological and are only suitable for solving engineering problems. Many fundamental mechanical problems remain open. By analyzing characteristics of internal state structure, we propose that granular matter is intrinsically multiscale, i.e. microscale of particle size, mesoscale of force chain, and macroscale of the bulk of granular matter. The correlations among difference scales would be crucial. The mesoscale force chain network is determined by both particle properties and macroscopic boundary conditions. The evolution of the force the chain network contributes to macroscopic mechanical properties of granular matter. In addition, we discuss the drawbacks in simplifying contact forces in the current models, and the difficulties in analyzing the interaction of interstitial fluid in wet granular matter. As an appropriate application of granular matter, debris flow can be studied with granular matter mechanics; meanwhile, debris flow brings more challenges which certainly motivate future studies on granular matter.

Keywords: Granular matter; Multiscale mechanics; Force chain; Contact mechanics; Debris flow

1. Introduction

The mechanics of granular matter studies the behaviour of particles when they are subjected to forces or displacements, and the subsequent effects on the environment. These particles are macroscopic and are characterized by a loss of energy whenever they interact, while thermal motion fluctuations can be ignored. Particle size distribution is very wide spanning from 1 μm to 10 m in particles such as nuts, coal, sand, rice, coffee, and corn flakes. Thus, granular matter is ubiquitous in nature, and may be one of the most manipulated materials in industry.

The study on granular matter could go back at least to C.A. de Coulomb (1736–1806), who first described the yielding of granular materials as a frictional process in 1773 [1]. From then on, most efforts have been made on the engineering practices of granular matter, rather than on the fundamental investigations. Only from the 1990s, some basic problems gradually attracted the attention of physicists and mechanists, since the mechanics of granular materials was found to present a spectrum of unique scientific challenges due to their self-organizing, fragile, and non-homogeneous flow behaviours. However, present theories of granular matter are only limited in a few ideal situations, such as quasi-static flow, rapid flow, and non-adhesion contact mechanics, as illustrated in Fig. 1, which have not yet touched the key problems of granular matter. Both quasi-static flow and rapid flow are relatively simple to analyze, but they are not observed in real life. All important granular flows are dense, such as hoppers, reasonably deep chute flows, and even commercial scale-vibrated boxes [1,2].

In dense granular matter, any particle experiences interactions with a finite number of other particles, each of which defines a different direction. Therefore, the local

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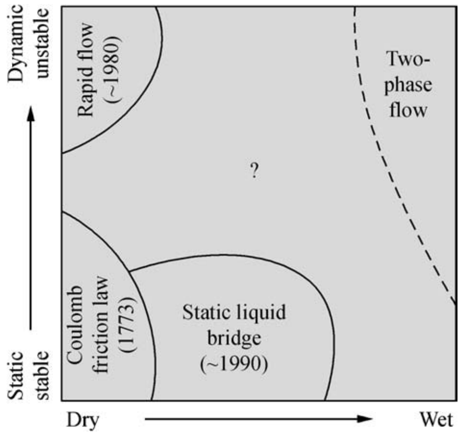

Fig. 1. Limited success of present theories for granular matter.

environment of a particle is not isotropic. The existence of the preferred directions on the particle scale implies the possible emergence of force chains, i.e. chains of contacts along which the forces are transmitted. It is well established that the structure of the force chains in granular materials plays a significant role in their overall constitutive behaviour. For example, granular materials are quite soft in the bulk. This is evidenced in their sound speed which is of the order of 100 m s^{-1}, roughly 50 times slower than the sound speed in their constituent solid material, indicating that the bulk granular material has an apparent elastic modulus due to deformation of force chains, more than three orders of magnitude smaller than its constituent solid. There are more internal state behaviours of dense granular materials arising from percolating force chain networks, such as jamming and arching, non-uniform convection and flow patterns, directional stress propagation, vibration-induced pattern formation, and the heterogeneity of the stress field.

Understanding of granular matter, somehow, is still at the infancy stage, and most phenomena cannot be well explained [3]. For example, continuum mechanical theory cannot be expected to describe these microscopic interparticle forces. The macroscopic field which is related to the microscopic forces is the stress. Even at small but finite spatial resolution, the stress tensor is determined by an appropriate average over forces, and it may not have any resemblance to the force distribution on the particle scale. Though the stress field is well defined on small scales, the constitutive relations correspond to continuum elasticity only on sufficiently large scales and thus cannot represent the large effect of disorder even on small scales.

From the scenario of internal state of granular matter revealed by photoelastic experiments and numerical simulations, we postulate that dense granular matter is intrinsically multiscale and multidisciplinary by its nature. The two end scales, i.e. the microscale corresponding to primary particle and the macroscale corresponding to the bulk of granular matter, are easy to understand. The mesoscale bridging microscale and macroscale would correspond to force chains. The characteristic times of the three scales have distinct physical meanings and differ greatly in value. For example, the contact time for a pair of approaching spheres is 10^{-5}–10^{-4} s; the lifetime of a force chain is about $1/\gamma$, where γ is the shear rate of the granular matter, and the transmitting time of a perturbation of an interaction through a force chain is about 10^{-7}–10^{-6} s. For example, if $\gamma = 10/\mathrm{s}$ as most frequently encountered in natural granular flows, a force chain would last 0.1 s. Once exceeding 0.1 s, the force chain would break and a new force chain would be established within 10^{-7}–10^{-6} s. Therefore, the force chain network always acts as a quasi-static structure. Therefore, we further propose that the force chain network is determined by both particle properties and macroscopic boundary conditions. The evolution of the force chain network contributes to macroscopic mechanical properties of granular matter.

In our research group, we preliminarily established an approach to correlate these three scales, and many efforts have been made for quantitative description of the force chain network. Some promising results have been obtained, which will be reported in this review article. The contact mechanics theories are introduced and compared, and drawbacks in simplifications in contact forces are discussed. The presence of interstitial non-Newtonian fluid on particles would be omnipresent in nature, and possible solutions to their interactions are proposed. The multiscale methodology for granular matter is also proposed, and some preliminary results are reported as well. The studies on granular matter would certainly promote the studies on debris flow, and detailed analysis about debris flow is presented.

2. Contact force and simplifications

Particles are usually irregular in shape, and have a wide size distribution from 1 μm to 10 m. For example, most pebbles along a beach have a smoothed ellipsoidal shape. The common aspect ratio is 7:6:3. For tiny particles around a few micros, surface attraction is dominant. The contact process is much more complicated than that of macroscopic particles [4]. Hertz firstly solved the contact problem for elastic spheres in the absence of adhesion, and Bradley solved it for rigid spheres with adhesion. Adhesion was first included in approximate theories of elastic contact by Johnson, Kendall, and Roberts (JKR) and by Derjaguin, Muller, and Toporov (DMT). Tabor showed that both JKR and DMT are limiting cases. Their relations are described in Fig. 2. The elastic parameter λ is defined as $\lambda \equiv \sigma_0 \sqrt[3]{9R/2\pi WE^*}$, where σ_0 is the stress at the equilibrium spacing, R and E^* the effective radius of curvature and modulus, respectively, W the adhesion energy, and P the applied load.

Analytical solutions of contact forces are often difficult to derive, and thus some necessary simplifications are usually required. The simplest model would be the linear model which introduces constant normal and tangential stiffness, known as the Hooke's law of elasticity. Another simplification model would be the soft-sphere model. The basic concept is that the contact process resembles a damping vibration by introducing new mechanical units, such as spring, dashpot, and slider, as shown in Fig. 3. We have to acknowledge that these simplifications greatly reduce the computational insensitivity, especially as computing interacting forces among a larger number of particles, but some hard physical problems are unavoidably introduced [6].

As the contact area increases when the normal force increases, geometric non-linearity occurs, and thus the linear model is not realistic. Moreover, the relative irreversible displacement of two particles in contact depends not only on the current contact force, but also on the previous loading history. For example, unlike Young's modulus and Poisson's ratio which are intrinsic parameters, friction is not intrinsic but depends slightly on other parameters such as the contact cleanliness, the presence of water, and the level of normal load. However, most work still use a constant value near 0.3. Especially, the majority of existing soft-sphere models requires some inputs that cannot be directly measured, such as spring constants and dashpot coefficients. The embarrassing problem would be that these parameters are just conceptual without realistic physical definition. The determination of these spring and dashpot parameters has been given very little attention in the literatures. The majority of efforts have simply assumed that the normal and tangential values are the same for the spring constant and coefficient of dissipation. Sometimes, the spring constant was made artificially small to reduce computational time which causes particles to be unrealistically soft.

Therefore, we claimed that any unrealistic simplifications would cause incorrect contact forces, force chain patterns and their evolutions, and thus result in false macroscopic mechanical properties [7,8]. In the fundamental studies on granular matter, contact forces must be computed with rigorous theories, such as Hertz, JKR, and DMT for appropriate regimes, as illustrated in Fig. 2. This viewpoint has already acted as the guideline in our group. For example, a software Tsinghua DEM Simulation (THDEM) was developed for dense granular mechanics studies where rigorous contact mechanics was employed. The potential value of THDEM is expected to enable examination of data that are normally inaccessible, and to perform rigorous parametric studies. Recently, THDEM was also used for some engineering applications, such as simulations of volcano Lahar in northeastern China, and some acceptable results about runout distance and impacting forces were obtained.

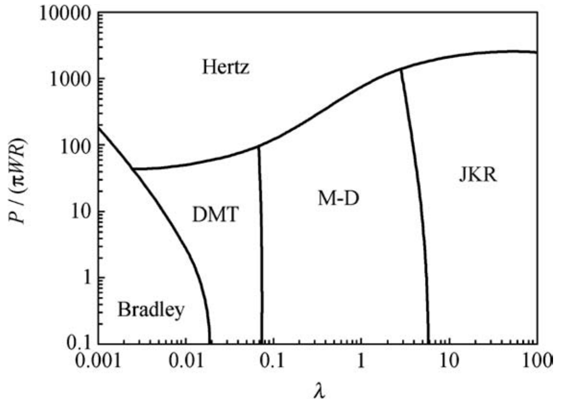

Fig. 2. Adhesion map reproduced from Ref. [5].

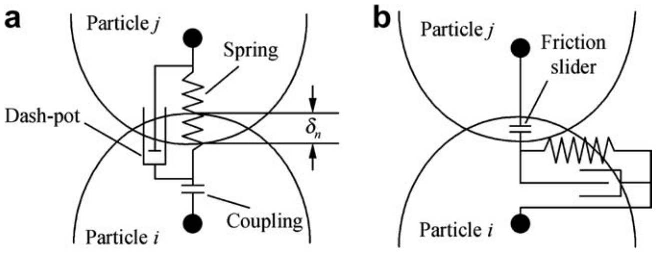

Fig. 3. Simplifications of contact forces in soft-sphere models (δ_n is the relative approach). (a) Normal force; (b) tangential force.

3. Interactions of interstitial fluid

Most studies on granular matter have focused on dry granular media, with no liquids between the grains. However, in geology and in many real world applications (e.g. food processing, pharmaceuticals, ceramics, civil engineering, construction, and many industrial applications), liquid is present between the grains [9]. This produces inter-grain cohesion and drastically modifies the mechanical properties of the granular media (e.g. the surface angle can be larger than 90°). It is well known that cohesion in wet granular materials depends on the amount of liquid in the system. As the liquid content is not great, a liquid bridge would be established, as schematically shown in Fig. 4. However, the analysis of both static and dynamic interactions of the interstitial fluid is rather rare [10].

For the normal relative motion of two spheres, the effect of wall slip on the squeeze flow could be analyzed by using the Reynolds lubrication approximation. The viscous force with a wall-slip boundary may be resolved to a non-slip solution by introducing a slip correction coefficient which may be related to the slip parameter, the flow index and the upper limit of integration. Generally, the wall slip could result in a reduction in the viscous force, and the reduction would increase as the flow index increases, suggesting that wall slip has a more profound effect on shear thickening material [12]. For the tangential relative motion, the velocity and the pressure equations could be derived, and thus the analytical solutions for the tangential force and the torque could be obtained. They will be much more complicated than that for a Newtonian fluid.

4. A multiscale approach based on force chain concept

Interparticle forces in granular matter form a heterogeneous distribution of force chains, which might be first

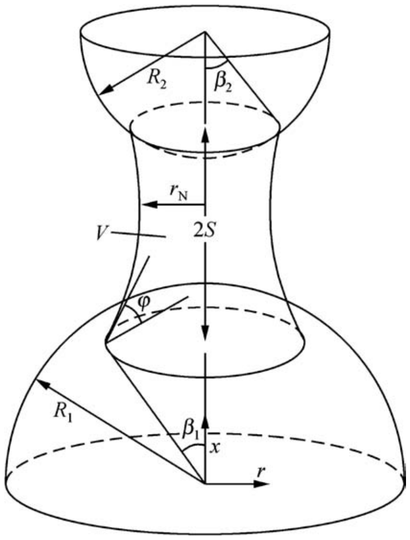

Fig. 4. A schematic diagram of a pendular liquid bridge of volume V and surface tension γ between two spheres of radii R_1 and R_2 separated by a distance $2S$ with a neck radius r_N, a liquid–solid contact angle φ, and half-filling angles β_1 and β_2 (reproduced from Ref. [11]).

termed as solid paths by Horne in 1965 [13]. Force chains are generally filament-like and parallel to the direction of loads, as shown in Fig. 5. The chain can only support loads along its own axis: successive contacts must be collinear, with the forces along the line of contacts, to prevent torques on particles within the chain. Neither friction at the contacts nor particle asperity can obviate this, though finite deformability allows small transverse loads to arise.

The typical length of force chains is around 10 grain diameters. They split and fuse at a variety of angles to form a network. Strong force chains where the contact force is higher are sparsely distributed throughout the granular matter, which are dominant to mechanical behaviours of granular matter. A large number of weak chains with lower contact forces exist in the vicinity of strong chains. They usually play an auxiliary role to the stability of strong force chains. Thus, one would conclude that the occurrence of the force chain network is an inherent feature of granular matter, which may substantially contribute to its specific characteristics.

Transmission of large forces strongly influences the resulting macroscopic stress. Understanding such forces and their spatial correlations, specifically in response to forces exerting at the system boundaries, represents a fundamental goal of granular matter mechanics. The problem is not only relevant to civil engineering and geophysics, but also to mechanics in explanations of jamming, shear-induced yielding, and mechanical responses.

In the past few years, a great number of theoretical and experimental studies have been devoted to this domain. However, solving practical engineering problems often requires the usage of phenomenological models, which introduce numerous parameters with no physical meaning. Moreover, a considerable number of constitutive models often contradict each other, notably in their basic concept [14]. Constitutive models for granular materials based on a micromechanical approach remain scarce. In our research group, we have proposed a multiscale approach to the mechanics of granular matter, as illustrated in Fig. 6. The microscale is related to the primary particle and the macroscale is about the granular matter bulk. The contact mechanics between primary particles acts as the basis for the multiscale approach, while the macroscale studies would be related to applications in engineering. Since deriving macroscopic stress equations from known microscopic or grain scale physics has proven to be quite difficult, a mesoscale, i.e. the force chain network, should be introduced and would play a vital role in bridging contact mechanics at microscale and macroscopic mechanical properties.

Generally, the dynamical system theory and statistical mechanics are the two major tools for solving multiscale problems, especially for cases of weak couplings or no couplings [15]. For example, the coupling between different scales is usually analyzed by assuming equal probability: the behaviours on macroscales are statistical averaging on microscale with a given probability density function. However, scale couplings in granular matter are nonlinear and strong so that they cannot be treated by either statistical methods or small perturbation any further [16]. For example, the microscale particle properties determine formation and stability of a force chain, while the interactions between force chains favor the macroscopic stress, as illustrated in Fig. 6. The unifying of the two correlations may eventually constitute a generalized theory for granular matter mechanics [17]. Due to the limit in experimental measures, the computer may be merely the last useful tool to investigate granular matter mechanics.

The software of THDEM developed in our group at Tsinghua University could be used for the micro/mesoscale correlation study. In our preliminary work on 2D dense

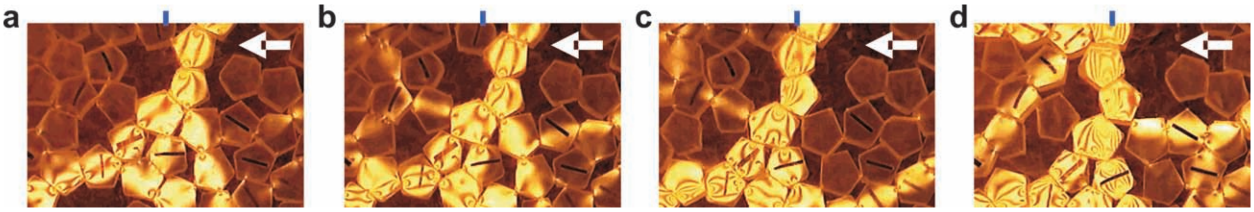

Fig. 5. Evolution of strong force chains in a shear cell, visualized by using the photoelastic technique. The top layer of particles slides from right to left as indicated by the white thick arrows.

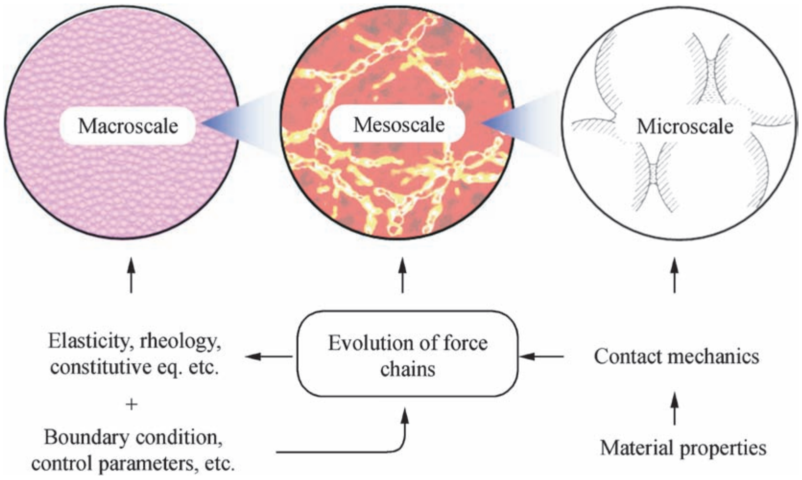

Fig. 6. A multiscale method for the study on granular matter.

granular matter with THDEM, we define force chains by a set of three plausible conditions: Grains *i*, *j*, and *k* are considered to be members of the same force chain if (1) particles *i*, *j* and *j*, *k* are next neighbors; (2) the pressure acting on each of the grains exceeds a certain threshold; (3) the connecting lines between *i*, *j* and *j*, *k* form an angle larger than 150°, i.e. the centers of three grains almost fall on a line. These three conditions were evaluated by a computer algorithm and it was found that most of the harder-stressed grains were part of the force chains, i.e. the main part of the static and dynamic pressure propagated along force chains. We exactly found that the pressure acting on a highly stressed grain inside a force chain was up to 100 times larger as compared with the average pressure of the neighboring grains not belonging to a force chain, i.e. the force distribution was strongly inhomogeneous.

For the meso/macroscale correlation study, we developed another model. A larger number of filaments were numerically constructed by placing straight rods of length in the two-dimensional simulation box with random center positions and randomly distributed orientations. Wherever two rods crossed, they were permanently cross-linked. These semiflexible filaments mimic force chains and are assumed to apply arbitrary constraint forces at the cross-link, but still allow the free rotation of the two filaments. Each filament has a compressing modulus and bending modulus. The sample with periodic boundary conditions was subjected to uniform deformations and demonstrated some of the novel elastic–plastic properties of such networks, as shown in Fig. 7. The calculated shear modulus and Young's modulus of this chain network will be compared with the previously published experimental data to improve the mesoscale study in our group. The fundamental feature of granular assemblies is that they are not able to undergo local tensile stress, which is due to their weakness. Thus, modeling the creation or the loss of contacts in given directions would be crucial in establishing a correct network.

Hence, it is appropriate to highlight some important issues.

(1) The multiscale in granular matter has specific characteristics. Huge difference exists in characteristic time scales, rather than length scales. For example, the contact time for a pair of approaching spheres is 10^{-5}–10^{-4} s; the lifetime of a force chain is about $1/\gamma$, where γ is the shear rate of the granular matter; local stress perturbation transmitting through a force chain takes about 10^{-7}–10^{-6} s.
(2) Particle surface friction is vital to the strength of the force chains, which would generate a greater tendency to form force chains, especially to form stronger force chains [1]. For example, the granular matter consisting of coarse primary particles usually exhibits higher shear modulus.
(3) The number density of micro fractures in solid is usually as many as $10^4/\text{mm}^2$, which is enough to make a statistical averaging. More importantly, these micro fractures develop individually before merging into a (or several) macro fracture(s). The force chain networks always cross-link with each other and the number of chains is not large enough to make reasonable averaging. Thus, granular matter mechanics is an exciting field with many fundamental problems required to be explored.

5. Mechanism of debris flow

Debris flows consist of concentrated mixtures of poorly sorted sediment and water, which can flow like liquids on sloping surfaces and can stop to form nearly rigid deposits. Debris flows constitute a significant natural hazard that can cause fatalities, damage structures, and diminish land productivity. Debris flows generally fall into three groups with respect to grain composition: water-stone flow or

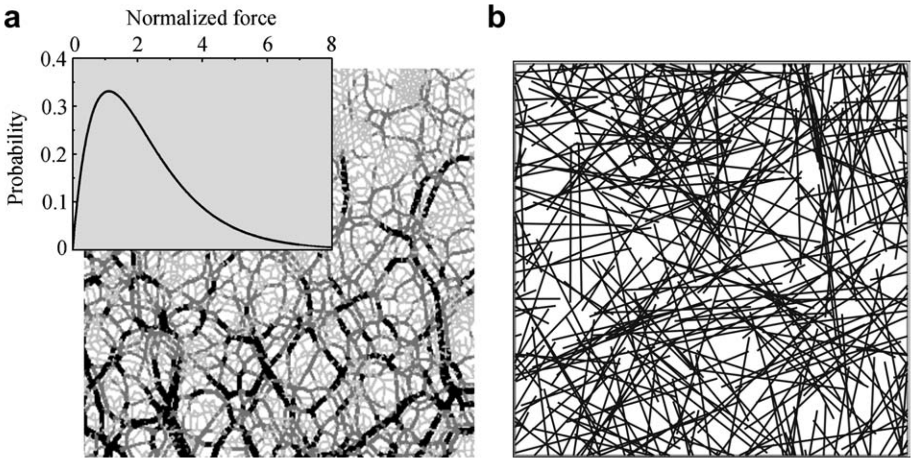

Fig. 7. Force chain network in a static packing of 2D granular materials. (a) Force chain network under gravitational environment with an inset showing the force distribution; (b) force chains mimiced with semiflexible cross-linked filaments.

sub-viscous debris flow, dominated by coarse grains; muddy flow, dominated by fine grains, and viscous debris flow composed of grains in a wide range. Most mechanical models of debris-flow deposition have been inspired by field observations and by small-scale laboratory experiments [18–20].

Measurements of pore-fluid pressure and total bed-normal stress at the base of several ~10 m experimental debris flows provide new insight into the process of debris-flow deposition [19]. As shown in Fig. 8, pore-fluid pressures nearly sufficient to cause liquefaction were developed and maintained during flow mobilization and acceleration, persisted in debris-flow interiors during flow deceleration and deposition, and dissipated significantly only during post-depositional sediment consolidation. In contrast, leading edges of debris flows exhibited little or no positive pore-fluid pressure. Therefore, debris-flow deposition results from grain-contact friction and bed friction concentrated along the flow perimeter, where high pore-fluid pressure is absent. Focused frictional resistance can occur in relatively homogeneous debris flows, but is enhanced if margins are composed predominantly of coarse clasts. These facts cannot be reasonably explained by using models that invoke widespread decay of excess pore-fluid pressure, intrinsic viscoplastic yield strength, or pervasive grain-collision stresses to explain debris-flow deposition. For example, because deposition results from frictional resistance focused on flow margins, deposit thickness cannot be used to infer intrinsic yield strength of moving debris.

At the front and the margin of a debris flow, coarse clasts are abundant, where the flow resistance is strongly concentrated as if the coarse snout acted as a moving dam that impeded the movement of more-fluid debris. The shear rate in debris flows is usually found to be less than 20/s, as reported in many experiments and field observations. Force chains must be established, as shown in Fig. 8(b). Evidence is that a dynamic loading such as vibration and crashing is required to break the force chain network during the initiation of debris flow. The macroscale rheological properties of dense granular matter, as men-

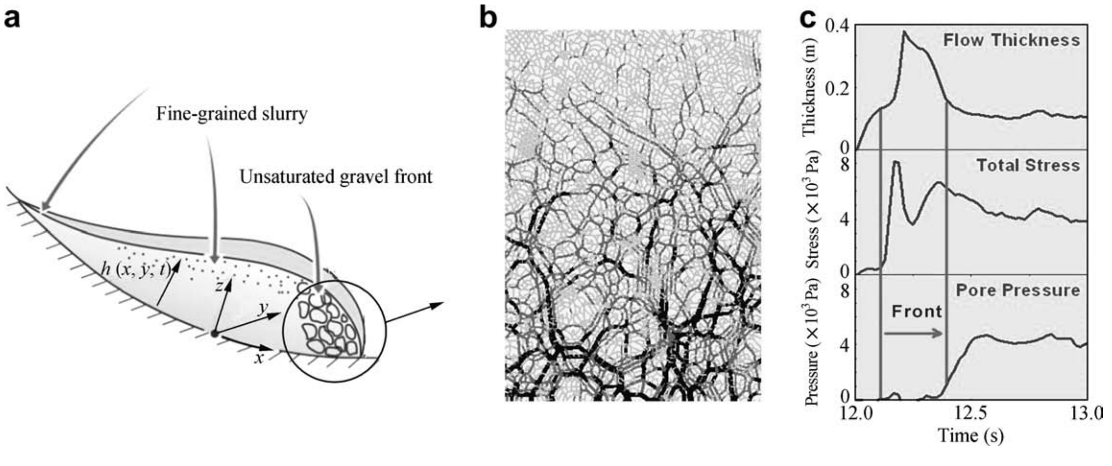

Fig. 8. The flow structure in a debris flow. (a) Segregation of particles resulting in coarse-graveled head and liquefied slurry basal consisting of rich fine particles; (b) presence of the force network in a surge head; (c) pore pressure variation along the debris flow.

tioned in Section 2, could be directly applied to describe behaviours of the surge head. For the remaining body of a debris flow, fine particles are fully suspended by water and then they could be well treated as uniform slurry. When coupling the surge head with debris body, overall behaviours of debris flow could be computed, including ignition, steady flow and deposit. Therefore, debris flow is one typical application of granular matter mechanics. Inversely, the specific problems encountered in debris flow motivate future studies on granular matter.

6. Perspectives

Granular matter is ubiquitous in nature and displays peculiar mechanical properties. Many observed phenomena are still not yet fully understood, and some fundamental mechanical problems remain open. Based on our understandings, we propose that granular matter is intrinsically multiscale and multidisciplinary, i.e. microscale of primary particle, mesoscale of force chain and macroscale of the bulk of granular matter. The force chain at mesoscale is determined by both granular properties and boundary conditions, and eventually contributes to mechanical properties of granular matters. Therefore, investigations on the force chain network would play an important role in the study in this domain. One of the typical applications of granular matter is the debris flow. On one hand, some fundamental study results could be directly applied for describing behaviours of debris flow. On the other hand, specific problems in debris flow would certainly prompt further studies on granular matter.

Acknowledgements

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